Alternating Sign Matrices, Polyhedra and Majorization

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Abstract

The theory of polyhedra and the theory of majorization are two areas of mathematics that have proved to be very useful in several mathematical areas. A purpose of this talk is to show how these areas play a role in the study of the class of alternating sign matrices. An *alternating sign matrix*, or ASM, is a $(0, \pm 1)$-matrix where the nonzero entries in each row and column alternate in sign, and where each row and column sum is 1. We study the convex cone generated by ASMs of order n, called the ASM cone, as well as several related cones and polytopes. Some decomposition results are shown, and we find a minimal Hilbert basis of the ASM cone. Also, a notion of sum-majorization is introduced and related to ASMs. The notion of (± 1)-doubly stochastic matrices and a generalization of ASMs are introduced and various properties are shown. For instance, we give a new short proof of the linear characterization of the ASM polytope, in fact for a more general polytope. Finally, we investigate faces of the ASM polytope, in particular edges associated with permutation matrices.

Joint work with:

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